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Nonsingular multidimensional cosmologies without fine tuning

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Abstract: Exact cosmological solutions for effective actions in D dimensions inspired by the tree-level superstring action are studied. For a certain range of free parameters existing in the model, nonsingular bouncing solutions are found. Among them, of particular interest can be open hyperbolic models, in which, without any fine tuning, the internal scale factor and the dilaton field (connected with string coupling in string theories) tend to constant values at late times. A cosmological singularity is avoided due to nonminimal dilaton-gravity coupling and, for $D > 11$, due to pure imaginary nature of the dilaton, which conforms to currently discussed unification models. The existence of such and similar solutions supports the opinion that the Universe had never undergone a stage driven by full-scale quantum gravity.

Keywords: p-branes, Classical Theories of Gravity, Cosmology of Theories beyond the SM, Physics of the Early Universe.

Contents

1. Introduction

It is widely recognized that the early Universe at sub-Planckian scales has been a scene for many strong effects which are nowadays either weak or even unobservable. Even remaining on a semiclassical level, one has to take into account the quantum properties of matter, the probable dynamical nature of extra dimensions predicted by modern unification theories (strings, supergravities, etc.), the existence of unusual kinds of matter such as strings and/or branes as well as modification of gravity at high energy densities and space-time curvatures.

Many of these features show the existence of semiclassical and even classical (tree-level) mechanisms able to circumvent the well-known singularity theorems and to prevent the formation of a cosmological singularity, keeping the curvature on sub-Planckian scales. This means that very probably there has not been an epoch of cosmological evolution driven by full-scale quantum gravity.

Thus, some nonsingular models have been built with the aid of gravitational lagrangians nonlinear in curvature (e.g., [1, 2]), whose origin may be explained by the quantum properties of matter fields. Supergravities and string theories, the candidate "theories of everything", also suggest new opportunities. Though, if the extra space-time dimensions, being an inevitable ingredient in such theories, are considered dynamically, the singularity problem becomes even more involved since, in addition to the usual cosmological scale factor, the extra dimensions can collapse or blow up, leading to a curvature singularity.

In this paper we will deal with particular multidimensional cosmologies obtainable from an effective action which conforms to the low energy limit of some currently discussed unification theories — see [3, 4] and references therein. (For recent reviews on string cosmologies see [5, 6].) This effective action admits as many as three natural mechanisms able to violate the usual energy conditions and hence potentially lead to nonsingular cosmological models. One mechanism is related to the nonminimal nature of the dilatonic scalar field and its interaction with antisymmetric form fields. Similar models in 4 dimensions have been discussed in ref. [7]. Second, the Brans-Dicke coupling constant ω may have values leading to a "wrong" sign of the dilaton kinetic term in the Einstein-frame lagrangian, which happens in dimensions larger than 11 [8] (in other terms, the dilaton becomes pure imaginary). Third, in some field models of string origin, formulated in space-times with multiple time coordinates, antisymmetric forms can have lagrangians with a "wrong" sign [4].

The models to be discussed are quite simple but natural from the viewpoint of the underlying theories. In a sense, our "vacuum" approach is alternative to that of ref. [9] where a hot brane gas is considered whereas global antisymmetric forms are ignored. We shall see that the first two mechanisms (but not the third one) really work and lead to globally regular cosmological solutions to the field equations.

Our aim here is only to demonstrate the effect of these mechanisms, therefore we do not consider the totality of exact solutions that might be obtained for the action (2.1) but restrict our attention to the simplest case: a single dilatonic field with a Brans-Dicke type lagrangian, a single antisymmetric form of axionic nature and a space-time with only two scale factors: external, $a(t)$, and internal, $b(t)$. We assume the external 3-dimensional space to be isotropic (spherical, flat or hyperbolic) and seek globally regular solutions such that, at late times, $a(t)$ exhibits expansion while $b(t)$ and the dilaton $\varphi(t)$ tend to finite constant values. We will specify the requirement to the "input" parameters of the theory that lead to the existence of models with the desired properties. We shall see that they exist among closed (spherical) and flat models only for special values of the integration constants, in other words, require fine tuning. Unlike that, favourable hyperbolic models appear without fine tuning in a certain range of the input and integration constants and therefore seem to be much more realistic.

There are many other arguments in favour of open cosmologies [10]. The observations are known to give the cosmological density factor Ω smaller or close to unity; meanwhile, the presently popular spatially flat cosmologies, most convenient for various calculations, require the precise equalty $\Omega = 1$, actually a sort of fine tuning. It is much more probable that the real Universe at least slightly violates this special requirement.

2. The model

Consider the action of D-dimensional gravity interacting with a dilatonic scalar field Φ and antisymmetric forms F_s, F_p, which account for contributions from both the Neveu-Schwarz — Neveu-Schwarz (NS-NS) and Ramond-Ramond (RR) sectors:

$$S_{\rm J} = \int d^D x \sqrt{g} \biggl\{ \Phi \biggl[\mathcal{R} - \omega \frac{(\partial\Phi)^2}{\Phi^2} - \sum_s \frac{\eta_s}{n_s!} F_s^2 \biggr] - \sum_r \frac{\eta_r}{n_r!} F_r^2 \biggr\} \tag{2.1}$$

where $\mathcal{R}$ is the scalar curvature, $g = |\det g_{MN}|$, $(\partial\Phi)^2 = g^{MN}\partial_M\Phi\partial_N\Phi$, $M, N = 0, \ldots, D-1$, ω is a (Brans-Dicke type) coupling constant, n_s and n_r are the ranks of antisymmetric forms belonging, respectively, to the NS-NS and RR sectors of the effective action; for each n-form, $F_n^2 = F_{n,M_1\ldots M_n}F_n^{M_1\ldots M_n}$; the sign factors $\eta_s = \pm 1$ and $\eta_r = \pm 1$ depend on a particular field model.

The action (2.1) is written in the so-called Jordan conformal frame where the field Φ is nonminimally coupled to gravity. This form is actually obtained in the weak field limit of many underlying theories as the framework describing the motion of fundamental objects, therefore we will interpret the metric g_{MN} appearing in (2.1) as the physical metric. Thus, if the fundamental objects are strings, one has in any dimension $\omega = -1$, while in cases where such objects are p-branes, one finds [3]

$$\omega = -\frac{(D-1)(p-1)-(p+1)^2}{(D-2)(p-1)-(p+1)^2}\,, \tag{2.2}$$

where p is the brane dimension and D is the space-time dimension. The NS-NS sector of string theory predicts a Kalb-Ramond type field with $n_s = 3$; the type IIA superstring effective action contains RR terms with $n_r = 2,\ 4$, while type IIB predicts $n_r = 3,\ 5$. The action (2.1) may also represent the bosonic sectors of theories like 11-dimensional supergravity (where the dilaton is absent, and there is a 4-form gauge field), or 10-dimensional supergravity (there is a dilaton and a 3-form gauge field), or 12-dimensional "field theory of F-theory" [8], admitting the bosonic sector of 11-dimensional supergravity as a truncation. The model [8] contains a dilaton and two F-forms of ranks 4 and 5; it admits electric 2- and 3-branes and magnetic 5- and 6-branes. The "wrong" sign $\eta_r = -1$ is found in IIA* and IIB* supergravities, obtained with timelike T-duality from IIB and IIA theories, respectively, and also in the field limit of M* theory, appearing as a strong coupling limit of IIA* theory [4, 11].

The standard transformation

$$g_{MN} = \Phi^{-2/(D-2)}\overline{g}_{MN} \tag{2.3}$$

leads to a theory reformulated in the Einstein conformal frame, more convenient for solving the field equations:

$$S_{\mathrm{E}} = \int d^Dx\sqrt{g_{\mathrm{E}}}\Big\{\overline{\mathcal{R}} - \eta_\omega(\partial\varphi)^2 - \sum_s\frac{\eta_s}{n_s!}\,\mathrm{e}^{2\lambda_s\varphi}F_s^2 - \sum_r\frac{\eta_r}{n_r!}\,\mathrm{e}^{2\lambda_r\varphi}F_r^2\Big\} \tag{2.4}$$

where all quantities are written in terms of the Einstein-frame metric $\overline{g}_{MN}$; $g_{\mathrm{E}} = |\det\overline{g}_{MN}|$; for the scalar field we have denoted

$$\Phi = \mathrm{e}^{\varphi/\omega_1}\,, \qquad \omega_1 = \sqrt{\left|\omega + \frac{D-1}{D-2}\right|}\,; \qquad \eta_\omega = \mathrm{sign}\left(\omega + \frac{D-1}{D-2}\right), \tag{2.5}$$

while the coupling constants λ_s and λ_r are

$$\begin{aligned} \lambda_s &= \frac{n_s-1}{\omega_1(D-2)} \qquad \text{(NS-NS sector)};\\ \lambda_r &= \frac{2n_r-D}{2\omega_1(D-2)} \qquad \text{(RR sector)}. \end{aligned} \tag{2.6}$$

D	p	ω	η_ω/ω_1^2	D	p	ω	η_ω/ω_1^2
any	1	-1	$D-2$	12	2	-2	$-10/9$
10	2	0	$8/9$	12	3	$-3/2$	$-5/2$
10	3	∞	0	12	4	$-8/5$	-2
10	4	2	$8/25$	12	5	-2	$-10/9$
10	5	0	$8/9$	12	6	-6	$-10/49$
10	6	$-4/9$	$72/49$	12	7	$1/2$	$5/8$
11	2	∞	0	12	8	$-4/11$	$110/81$
11	3	-2	$-9/8$	14	2	$-4/3$	-4
11	4	$-5/2$	$-18/25$	14	6	$-16/11$	$-132/49$
11	5	∞	0	26	3	$-17/16$	-48
11	6	$1/4$	$36/49$	26	4	$-50/47$	$-1128/25$

Table 1: Values of ω and η_ω/ω_1^2 for different space-time and brane dimensions.

The sign factor η_ω distinguishes "normal" theories ($\eta_\omega = +1$), such that the kinetic term of the φ field in (2.4) has the normal sign corresponding to positive energy, from anomalous theories where this sign is "wrong" ($\eta_\omega = -1$). It should be noted that many theories with $D > 11$ involve $\eta_\omega = -1$. According to (2.5),

$$\frac{\eta_\omega}{\omega_1^2} = (D-2)\left[1 - \frac{(D-2)(p-1)}{(p+1)^2}\right]. \tag{2.7}$$

Evidently, under the condition $(D-2)(p-1) > (p+1)^2$ we have $\eta_\omega = -1$. For $p = 2,\ 5$ this happens when $D > 11$, and for $p = 3,\ 4$ when $D > 10$.

Table 1 gives the values of ω and η_ω/ω_1^2 for some particular space-time and brane dimensions.

Some comments are in order. First, the well-known result $\omega = -1$ for strings ($p = 1$) in any dimension is recovered. Second, one obtains $\omega = \infty$ for 2- and 5-branes in 11 dimensions, which conforms to the absence of a dilaton in 11D supergravity that predicts such branes. Third, in 12 dimensions one has $\eta_\omega = -1$ for $p < 7$, and such a theory ref. [8] does contain a pure imaginary dilaton: the F-forms of ranks 4 and 5 are coupled to a dilaton field φ with the coupling constants $\lambda_1^2 = -1/10$ and $\lambda_2 = -\lambda_1$, respectively, while the product $\lambda\varphi$ is real. As is concluded in ref. [8], for $D > 11$ "imaginary couplings are exactly what is needed in order to make a consistent truncation to the fields of type IIB supergravity possible". In our (equivalent) formulation, φ and λ are real and the unusual nature of the coupling is reflected in the sign factor η_ω.

Supersymmetric models with $D = 14$ are also discussed [12, 13], while $D = 26$ is the well-known dimension for bosonic strings.

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3. Solutions

Let us specify the space-time structure and the Einstein-frame metric as

$$\mathbb{M} = \mathbb{R}_u \times \mathbb{M}_0 \times \mathbb{M}_1 \times \cdots \times \mathbb{M}_n\,, \qquad \dim \mathbb{M}_i = d_i\,, \tag{3.1}$$

$$ds_{\rm E}^2 = -\,{\rm e}^{2\alpha(u)} du^2 + \sum_{i=0}^{n} {\rm e}^{2\beta^i(u)} ds_i^2 \tag{3.2}$$

where u is a time coordinate ranging in $\mathbb{R}_u \subset \mathbb{R}$ and ds_i^2 are the u-independent metrics of the factor spaces $\mathbb{M}_i$, assumed to be Ricci-flat for $i = 1, \ldots, n$ whereas ds_0^2 in $\mathbb{M}_0$ describes a space of constant curvature $K_0 = 0, \pm 1$, corresponding to the three types of isotropic spaces; $\mathbb{M}_0$ is thus interpreted as an external (observed) factor space.

There is a diversity of exact solutions for the action (2.4) without L_m in space-times like (3.2), discussed, in particular, in refs. [14, 15, 16] (see also references therein). We will be only interested here in cosmological solutions for a very simple special case: a single antisymmetric form $F_{[d_0]}$ from the NS-NS or RR sector, having a single (up to permutations) nontrivial component $F_{1\ldots d_0}$ where the indices refer to $\mathbb{M}_0$, and a single internal space $\mathbb{M}_1$, so that in (3.2) $i = 0, 1$, and $\varphi = \varphi(u)$. Then the field equations are easily integrated.

Let u be a harmonic time coordinate for the metric (3.2), so that $\alpha = d_0\beta^0 + d_1\beta^1$.

The F-form is magnetic-type; the Maxwell-like equations due to (2.4) are satisfied trivially while the Bianchi identity $dF = 0$ implies

$$F_{1\ldots d_0} = Q\sqrt{g_0}\,, \qquad Q = \text{const.} \tag{3.3}$$

where g_0 is the metric determinant corresponding to ds_0^2 and Q is a charge, to be called the *axionic charge* since the only nonzero component of F can be represented in terms of a pseudoscalar axion field in $d_0 + 1$ dimensions. The remaining unknowns are β^0, β^1 and φ.

In the Einstein equations $\overline{\mathcal{R}}_M^N - \frac{1}{2}\delta_M^N \overline{\mathcal{R}} = T_M^N$, written for the Einstein-frame metric (3.2), the stress-energy tensor T_M^N has the form

$${\rm e}^{2\alpha} T_M^N = -\frac{1}{2}\eta_F Q^2\, {\rm e}^{2d_1\beta^1 + 2\lambda\varphi}\, {\rm diag}\,(+1,\ [-1]_{d_0},\ [+1]_{d_1}) - \frac{1}{2}\eta_\omega \dot\varphi^2\, {\rm diag}\,(+1,\ [-1]_{d_0+d_1}) \tag{3.4}$$

where the first place on the diagonal belongs to u and the symbol $[f]_d$ means f repeated d times; $\eta_F = \pm 1$ is the sign factor of our F-form, originating from η_s or η_r in (2.1) or (2.4).

Due to the EMT property $T_u^u + T_z^z = 0$ (where z belongs to $\mathbb{M}_0$), the corresponding Einstein equation has the Liouville form $\ddot\alpha - \ddot\beta^0 + K_0(d_0-1)^2\, {\rm e}^{2\alpha - 2\beta^0}$, whence

$$\frac{1}{d_0 - 1}\, {\rm e}^{\beta^0 - \alpha} = S(-K_0, k, u) \stackrel{\rm def}{=} \begin{cases} {\rm e}^{ku}\,, & K_0 = 0\,, \quad k \in \mathbb{R}\,; \\ k^{-1}\cosh ku\,, & K_0 = 1\,, \quad k > 0\,; \\ k^{-1}\sinh ku\,, & K_0 = -1\,, \quad k > 0\,; \\ u\,, & K_0 = -1\,, \quad k = 0\,; \\ k^{-1}\sin ku\,, & K_0 = -1\,, \quad k < 0\,, \end{cases} \tag{3.5}$$

where k is an integration constant and one more constant is suppressed by a proper choice of the origin of u. Equation (3.5) can be used to express β^0 in terms of β^1.

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It is helpful to consider the remaining unknowns as a vector $x^A = (\beta^1,\ \varphi)$ in the 2-dimensional target space $\mathbb{V}$ with the metric

$$(G_{AB}) = \begin{pmatrix} d\,d_1 & 0 \\ 0 & \eta_\omega \end{pmatrix}, \qquad (G^{AB}) = \begin{pmatrix} 1/(d\,d_1) & 0 \\ 0 & \eta_\omega \end{pmatrix}, \qquad d \stackrel{\rm def}{=} \frac{D-2}{d_0-1}. \tag{3.6}$$

The equations of motion then take the form

$$\ddot{x}^A = -\eta_F Q^2 Y^A \mathrm{e}^{2y} \tag{3.7}$$

$$G_{AB}\dot{x}^A\dot{x}^B + \eta_F Q^2\,\mathrm{e}^{2y} = \frac{d_0}{d_0-1}K, \qquad K = \begin{cases} k^2\,\mathrm{sign}\,k, & K_0=-1\,, \\ k^2, & K_0 = 0,+1\,. \end{cases} \tag{3.8}$$

with the function $y(u) = d_1\beta^1 + \lambda\varphi$, representable as a scalar product of x^A and the constant vector $\vec{Y}$ in $\mathbb{V}$:

$$y(u) = Y_A x^A, \qquad Y_A = (d_1,\,\lambda), \qquad Y^A = \left(\frac{1}{d}, \eta_\omega\lambda\right). \tag{3.9}$$

Equation (3.8) is a first integral of (3.7) that follows from the $\binom{u}{u}$ component of the Einstein equations.

The simplest solution corresponds to $Q=0$ (scalar vacuum):

$$\beta^1 = c^1 u + \underline{c}^1, \qquad \varphi = c_\varphi u + \underline{c}_\varphi\,, \tag{3.10}$$

where c^1, $\underline{c}^1$, c_φ and $\underline{c}_\varphi$ are integration constants. Due to (3.8), the constants $c^A = (c^1,\,c_\varphi)$ are related by

$$c_A c^A = dd_1(c^1)^2 + \eta_\omega c_\varphi^2 = \frac{d_0}{d_0-1}K\,. \tag{3.11}$$

If $Q\neq 0$, eqs. (3.7) combine to yield an easily solvable (Liouville) equation for $y(u)$:

$$\ddot{y} + \eta_F Q^2 Y^2\,\mathrm{e}^{2y} = 0\,, \qquad Y^2 = Y_A Y^A = \frac{d_1}{d} + \eta_\omega\lambda^2\,. \tag{3.12}$$

This is a special integrable case of the equations considered, e.g., in refs. [14, 15, 16].

Among the diverse solutions to (3.12) existing for different values of Y^2, we will choose, for our purposes, the solution for $Y^2>0$. One of the reasons is that even for $\eta_\omega=-1$ one has $Y^2>0$ for fields from the NS-NS sector in any dimension and for fields from the RR sector if $D<17$. For $Y^2>0$, eq. (3.12) gives

$$\mathrm{e}^{-y(u)} = \frac{|Q|Y}{h}\cosh[h(u+u_1)] \tag{3.13}$$

where $Y = |Y^2|^{1/2}$, $h>0$ and u_1 are integration constants. The unknowns x^A are expressed in terms of y as follows:

$$x^A = \frac{Y^A}{Y^2}y(u) + c^A u + \underline{c}^A \tag{3.14}$$

where the constants $c^A = (c^1,\,c_\varphi$ and $\underline{c}^A = (\underline{c}^1,\ \underline{c}_\varphi)$ satisfy the orthogonality relations

$$c^A Y_A = 0\,, \qquad \underline{c}^A Y_A = 0\,. \tag{3.15}$$

Finally, the constraint (3.8) leads to one more relation among the constants:

$$\frac{h^2}{Y^2} + c_A c^A = \frac{d_0}{d_0-1}K\,. \tag{3.16}$$

4. Analysis of cosmological models

4.1 Prelimaries

In what follows, we put $d_0 = 3$, so that $d_1 = D - 4$, and identify, term by term, the Jordan-frame metric $ds_{\rm J}^2$ obtained in the above notations (2.3), (3.2),

$$ds_{\rm J}^2 = \exp\left[-\frac{2\varphi}{\omega_1(D-2)}\right]\left\{\frac{e^{-d_1\beta^1}}{2S(-K_0,k,u)}\left[\frac{-du^2}{4S^2(-K_0,k,u)} + ds_0^2\right] + e^{2\beta^1} ds_1^2\right\}, \qquad (4.1)$$

where the function $S(.,.,.)$ is defined in (3.5), with the familiar form of the metric

$$ds_{\rm J}^2 = -dt^2 + a^2(t)ds_0^2 + b^2(t)ds_1^2\,, \qquad (4.2)$$

so that $a(t)$ and $b(t)$ are the external and internal scale factors and t is the cosmic time.

To select nonsingular models, let us use the Kretschmann scalar $\mathcal{K} = R_{MNPQ}R^{MNPQ}$, which is in our case a sum (with positive coefficients) of squares of all Riemann tensor components $R_{MN}{}^{PQ}$. Thus as long as $\mathcal{K}$ is finite, all algebraic curvature invariants of this metric are finite as well. For the metric (4.2) with $d_0 = 3$ one has (the primes denote d/dt):

$$\mathcal{K} = 4\left[3\left(\frac{a''}{a}\right)^2 + d_1\left(\frac{b''}{b}\right)^2 + 3d_1\left(\frac{a'b'}{ab}\right)^2\right] + 2\left[6\left(\frac{K_0 + a'^2}{a^2}\right)^2 + d_1(d_1-1)\,\frac{b'^4}{b^4}\right]. \qquad (4.3)$$

By (4.3), $\mathcal{K} \to \infty$ and hence the space-time is singular when $a \to 0$, $a \to \infty$, $b \to 0$ or $b \to \infty$ at finite proper time t. Accordingly, our interest will be in the asymptotic behaviour of the solutions at both ends of the range $\mathbb{R}_u = (u_{\min},\ u_{\max})$ of the time coordinate u, defined as the range where both a^2 and b^2 in (4.2) are regular and positive. (Note that, as $t \to \pm\infty$, a singularity does not occur when $b(t) \to 0$, or $a \to 0$ in case $K_0 = 0$.) At any $u \in \mathbb{R}_u$ all the relevant functions are manifestly finite and analytical. The boundary values $u_{\max}$ and $u_{\min}$ may be finite or infinite; a finite value of $u_{\max}$ or $u_{\min}$ coincides with a zero of the function (3.5).

Among regular solutions, of utmost interest are those in which $a(t)$ grows while $b(t)$ tends to a finite constant value as $t \to \infty$. Any asymptotic may on equal grounds refer to the evolution beginning or end due to the time-reversal invariance of the field equations. We will for certainty speak of expansion or inflation, bearing in mind that the same asymptotic may mean contraction (deflation).

Let us now enumerate the possible kinds of asymptotics.

- *Type I:* $u \to \pm\infty$, where

$$dt^2 \sim e^{(A-2k)|u|}\,, \qquad a^2 \sim e^{A|u|}\,, \qquad b^2 \sim e^{B|u|}\,, \qquad (4.4)$$

  with $k > 0$ and certain constants A and B depending on the solution parameters. A favourable asymptotic of $a(t)$ takes place when $A \geq 2k$:

  (i) $A > 2k:$ $\quad t \to \infty\,, \quad a \sim t^{A/(A-2k)}$ (power-law inflation);

  (ii) $A = 2k:$ $\quad t \sim |u| \to \infty\,, \quad a \sim e^{kt}$ (exponential inflation).

A reformulation for $k < 0$ is evident. The internal scale factor $b(t)$ tends to a finite limit if $B = 0$, i.e., under a special condition on the model parameters (fine tuning).

- *Type Ia:* a modification of type I when $k = 0$, so that at $u \to \infty$

$$dt^2 \sim u^{-3}\,\mathrm{e}^{Au} du^2\,, \qquad a^2 \sim u^{-1}\,\mathrm{e}^{Au} du^2\,, \qquad b^2 \sim \mathrm{e}^{Bu} \tag{4.5}$$

If $A > 0$, we have, as desired, $t \to \infty$ and $a \to \infty$; the expansion may be called "slow inflation" since it is only slightly quicker than linear: the derivative $da/dt \sim u$, which behaves somewhat like $\ln t$. If $A \leq 0$, then $a \to 0$ at finite t (singularity). As for $b(t)$, one may repeat what was said in case I.

- *Type II:* $u \to 0$, where the function (3.5) tends to zero, so that $S(-K_0, k, u) \sim u$, while other quantities involved are finite. In this case

$$dt^2 \sim \frac{1}{u^3}\,, \qquad a^2 \sim \frac{1}{u}\,, \qquad b^2 \to \text{fin}\,. \tag{4.6}$$

According to (4.6), $t \to \pm\infty$, $a(t) \sim |t|$ (linear expansion or contraction), whereas both $b(t)$ and $\varphi(t)$ tend to finite limits since they do not depend on $S(-K_0, k, u)$.

The dilaton φ in all cases behaves like $\ln b(t)$, but, in general, with another constant B in each particular solution.

This exhausts the possible kinds of asymptotics for $Y^2 > 0$. Solutions with $Y^2 \leq 0$, which can emerge when $\eta_\omega = -1$ and/or with $\eta_F = -1$, may have other asymptotics, but they are of lesser interest.

4.2 Scalar-vacuum cosmologies

The scalar-vacuum models (4.1), (3.10) depend on two input constants, D (or $d_1 = D - 4$, or $d = (D-2)/2$) and ω (or ω_1) and three integration constants k, c^1, c_φ related by (3.11); two more constants, $\underline{c}^1$ and $\underline{c}_\varphi$, only shift the scales in $\mathbb{M}_1$ and along the φ axis and do not affect the qualitative behaviour of the models.

Closed models, $K_0 = +1$. In this case in (4.1) $S = k/\cosh ku$, $k > 0$, hence the solution has two type I asymptotics at $u \to \pm\infty$, with $k > 0$ and the following constants $A = A_\pm$:

$$A_\pm = -k \mp \left[d_1 c^1 + \frac{c_\varphi}{d\omega_1}\right], \tag{4.7}$$

so that at least at one of the asymptotics $A < 0$ whence $a \to 0$ at finite t, a singularity. It is also easily seen that if $b(t) \neq$ const., it behaves as e^{Bu}, $B =$ const, and tends to zero at one of the limits $u \to \pm\infty$.

Spatially flat models, $K_0 = 0$. One has simply

$$a^2(t) = \mathrm{e}^{Au}\,, \qquad dt \sim \mathrm{e}^{(A-2k)u/2} du\,, \tag{4.8}$$

where $A = -c_\varphi/(d\omega_1) - d_1 c^1 - k$, $k \in \mathbb{R}$, and again $b^2(t) = \mathrm{e}^{Bu}$, $B =$ const. Thus each of the scale factors is either constant, or evolves between zero and infinity, and $a = 0$ occurs at finite t.

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Hyperbolic models, $K_0 = -1$. If $k > 0$ [note that, when $\eta_\omega = 1$, there is necessarily $k > 0$ due to (3.11)], one has in (4.1) $S = k^{-1} \sinh ku$. Hence the model evolves between a type I asymptotic at $u \to \infty$, with A coinciding with A_+ in eq. (4.7), and type II at $u = 0$. Since type II is regular, a necessary condition for having a nonsingular model is $A \geq 2k$.

To find out if and when it happens for $\eta_\omega = +1$, it is convenient to introduce, instead of the two constants c^1 and c_φ connected by (3.11), an "angle" θ such that

$$-c^1 = \sqrt{\frac{3}{2dd_1}} k \cos\theta \,, \qquad -c_\varphi = \sqrt{\frac{3}{2}} k \sin\theta \,. \tag{4.9}$$

The condition $A \geq 2k$ will be realized for a certain choice of the integration constants if A_+ given by (4.7) has, as a function of θ, a maximum no smaller than $2k$. An inspection shows that it happens if

$$\omega_1^2 \leq \frac{1}{[d(6d - d_1)]} = \frac{1}{[(D-1)(D-2)]} \,. \tag{4.10}$$

This is the only example of a nonsingular (bouncing) vacuum model with $\eta_\omega = +1$.

In case $k > 0$, $\eta_\omega = -1$, a choice of c_φ and c^1 subject to (3.11) such that $A > 2k$ is easily made for any ω_1.

For $\eta_\omega = -1$, $k = 0$, the model evolves between type Ia and II asymptotics, where at the Ia end ($u \to \infty$)

$$A = -d_1 c^1 - \frac{c_\varphi}{(d\omega_1)} \,, \qquad B = 2c^1 - \frac{c_\varphi}{(d\omega_1)} \,. \tag{4.11}$$

The necessary condition for regularity, $A > 0$, is satisfied for proper c_1 and c_φ which can be chosen without problems.

In case $\eta_\omega = -1$, $k < 0$, the function (3.5) is simply $|k|^{-1} \sin |k| u$, and the model has two type II asymptotics at adjacent zeros of S, say, $u = 0$ and $u = \pi/|k|$. This model is automatically nonsingular for any further choice of integration constants.

We conclude that among vacuum models only some hyperbolic ones can be nonsingular. For $\eta_\omega = +1$ in such a case $a(t)$ evolves from linear decrease to inflation, or from deflation to linear growth. Only in the latter case both $b(t)$ and φ tend to finite limits as $t \to \infty$ without any fine tuning.

For $\eta_\omega = -1$ there is a model interpolating between two asymptotics of the latter kind. Thus, as t changes from $-\infty$ to $+\infty$, $a(t)$ bounces from linear decrease to linear increase (generically with a different slope) whereas $b(t)$ and $\varphi(t)$ smoothly change from one finite value to another. The latter model exists for generic values of the integration constants.

4.3 Cosmologies with an axionic charge

The solution contains, in addition to the input parameters D, ω and λ, three independent essential integration constants: the "scale parameter" k, the charge Q and also h and c_φ connected by (3.16); the constant c^1 is excluded by the first relation (3.15)

$$d_1 c^1 + \lambda c_\varphi = 0 \tag{4.12}$$

so that the quantity $c^A c_A$, appearing in (3.16), is expressed as $c^A c_A = \eta_\omega (d/d_1) c_\varphi^2 Y^2$. The fourth constant, the "shift parameter" u_1, as well as $\underline{c}^1$ and $\underline{c}_\varphi$, connected by (3.15), are qualitatively inessential.

Let us begin with "normal" models, $\eta_\omega = +1$. The solution (3.14) has the form

$$\beta^1(u) = \frac{1}{dY^2} y(u) + c^1 u + \underline{c}^1\,, \tag{4.13}$$

$$\varphi(u) = \frac{\lambda}{Y^2} y(u) + c_\varphi u + \underline{c}_\varphi\,. \tag{4.14}$$

with $y(u)$ given by (3.13). The form of (3.16) implies $k > 0$ and suggests a notation similar to (4.9), namely,

$$h = \sqrt{\frac{3}{2}} kY \cos\theta\,, \qquad c_\varphi = \left(1 + \lambda^2 \frac{d}{d_1}\right)^{-\frac{1}{2}} \sqrt{\frac{3}{2}} k \sin\theta\,. \tag{4.15}$$

Let us now pass to the asymptotic description.

Common asymptotic $u \to \infty$. This asymptotic is common to all K_0 and belongs to type I with

$$A = -k + hA_1 + c_\varphi A_2\,, \qquad B = hB_1 + c_\varphi B_2 \tag{4.16}$$

with the notations

$$A_1 = \frac{1}{dY^2}\left(d_1 + \frac{\lambda\eta_\omega}{\omega_1}\right), \qquad A_2 = \lambda - \frac{1}{d\omega_1}\,,$$
$$B_1 = \frac{1}{dY^2}\left(-2 + \frac{\lambda\eta_\omega}{\omega_1}\right), \qquad B_2 = -\left(\frac{2\lambda}{d_1} + \frac{1}{d\omega_1}\right). \tag{4.17}$$

The condition $A \geq 2k$, or

$$hA_1 + c_\varphi A_2 \geq 3k\,, \tag{4.18}$$

required for nonsingular models, may be realized for small ω_1. Using the representation (4.15), one can find, precisely as in the scalar-vacuum case, a condition under which $A_{\max}$, the maximum value of A as a function of θ, satisfies $A_{\max} \geq 2k$. Curiously, the coupling λ drops away from the calculation, and the resulting condition coincides with (4.10).

Meanwhile, B can have any sign, therefore the asymptotic of $b(t)$ is uncertain; however, by choosing the ratio c_φ/h (that is, by fine-tuning θ) one can achieve $B = 0$, so that $b(t)$ has a finite limit.

The solution behaviour at the other end of the range $\mathbb{R}_u$ depends on K_0.

Closed models. For $K_0 = +1$, $u \to \infty$ the asymptotic behaviour is

$$a^2(t) \sim \mathrm{e}^{A'|u|}\,, \qquad b^2(t) \sim \mathrm{e}^{B'|u|}\,, \qquad dt \sim \mathrm{e}^{(A'-2k)|u|/2} du \tag{4.19}$$

where A' and B' coincide with A and B given by (4.16) with the replacement $c_\varphi \to -c_\varphi$ (or $\theta \to -\theta$). Therefore the description is the same up to the replacement $t \to -t$. When simultaneously $A \geq 2k$ and $A' \geq 2k$, one has a deflation $\to$ inflation transition for the scale factor $a(t)$, with generically different powers $A/(A-2k)$ and $A'/(A'-2k)$ at the contraction and expansion phases, and a regular bounce between them; at one end the evolution may be exponential. But, even if $b(t) \to$ fin as $t \to \infty$, the evolution of $b(t)$ begins with $b = 0$ or $b = \infty$ unless the model parameters are further fine-tuned.

Spatially flat models. The case $K_0 = 0$ differs from $K_0 = +1$ in that k can have either sign, and when comparing the asymptotics $u \to \infty$ and $u \to -\infty$, one has to replace $k \to -k$. Hence, if we deal with a solution such that the Universe evolves in a power-law inflationary regime in the asymptotic $u \to \infty$, then, in the asymptotic $u \to -\infty$, we find a behaviour like $a(t) \sim |t|^s$, $s < 1$. So in the remote past there might be a subluminal contraction while in the remote future the model inflates. A necessary condition for this type of behaviour is again (4.18) (with k replaced by $|k|$), leading to the requirement (4.10).

Concerning $b(t)$, one only can repeat what was said about $K = +1$.

Hyperbolic models. The second asymptotic, $u \to 0$, is type II. Thus, provided $A > 2k$ (see (4.16)), there occurs linear contraction of the Universe in the remote past and inflation in the remote future, or, in a time-reversed model, deflation in the remote past and linear expansion in the future. The latter opportunity seems especially attractive since both φ (hence the string coupling) and $b(t)$ (hence the effective gravitational constant) tend to finite limits *automatically*, without any need for fine tuning.

We see that even models where the kinetic terms of φ and F_{MNP} have both normal signs ($\eta_\omega = \eta_F = 1$), predict some nonsingular bouncing cosmologies.

"Anomalous" models with $\eta_\omega = -1$, like their vacuum counterparts, bear some new features compared to $\eta_\omega = +1$, connected with the relation (3.16). Namely, eqs. (3.13), (4.13), (4.14) are still valid, but now we can have $c^A c_A \leq 0$. Therefore, first, the representation (4.15) is no more valid, thus cancelling the restriction (4.10). Second, for $K_0 = -1$, one can have now solutions with $k \leq 0$, with a different analytical form and different behaviour.

As a result of the first of these circumstances, one obtains a bouncing behaviour of $a(t)$ in closed ($K_0 = 1$) and flat ($K_0 = 0$) models, having two type I asymptotics as described above, by simply choosing the integration constants h and φ_1 from a proper range, without further restrictions on the input parameters (provided they lead to $\eta_\omega = -1$). However, fine tuning is still necessary for obtaining finite limits of $b(t)$ and $\varphi(t)$ at late times.

The appearing hyperbolic models with $k = 0$ interpolate between type II ($u \to 0$) and type Ia ($u \to \infty$) asymptotics (linear contraction $\to$ slow inflation or slow deflation $\to$ linear expansion), under the condition

$$A = hA_1 + c_\varphi A_2 > 0\,, \tag{4.20}$$

easily satisfied by choosing proper h and c_φ.

There also appear bouncing models with $k < 0$ which qualitatively behave quite similarly to their vacuum counterparts, interpolating between two type II asymptotics.

5. Concluding remarks

The bouncing mechanism discussed here works in a certain range of the free parameter of the model, the Brans-Dicke type constant ω. Part of this favourable range, see eq. (4.10), corresponds to "normal" dilatonic fields with positive energy in the Einstein frame, and such bouncing models exist for all $D > 4$. If we, however, ascribe the origin of ω to fundamental p-branes in the spirit of ref. [3], it turns out that only "anomalous" theories with $\eta_\omega = -1$ lead to bouncing models. [Indeed, in case $\eta_\omega = 1$ eq. (2.7) gives $\omega_1^{-2} \le D-2$, contrary to (4.10).] Moreover, such models only exist for $D > 11$, see the table in section 2 and the comments after it.

The existence of a type II asymptotic of open models, making it possible to avoid fine tuning in getting a desired large t asymptotic, is quite a general phenomenon for cosmologies with the structure (3.1). It actually follows from a "stiff" character of the stress-energy tensor (pressure in the external space is equal to energy density) that leads to eq. (3.5) and is common to the dilaton and the axion in the Einstein frame.

The present nonsingular vacuum and axionic models evidently cannot pretend to describe the full-time evolution of the Universe but rather the epoch of maximum contraction under the assumption that this stage is dominated by scalar-vacuum and axionic effects. Unlike other scenarios with a bounce at sub-Planckian scales, such as the brane gas [2, 9] and Pre-Big Bang [6, 17] scenarios, no higher-order curvature terms are needed here to complement the model. Furthermore, our models, with $b(t)$ and $\varphi(t)$ tending to finite constant values, become effectively 4-dimensional at late times, with constant values of the effective gravitational constant and string tension, so the further evolution can be studied using conventional methods, in particular, an inflationary period can follow. Though, in our view, inflation is mostly needed for solving the problems of Big Bang cosmology emerging due to the existence of multiple causally disconnected regions, whereas bouncing cosmologies do not create such problems.

In any bouncing model a separate problem is the origin of its initial state. We would note here that in some cases, as $t \to -\infty$, the dilaton field $\Phi = \mathrm{e}^{-\varphi/\omega_1}$ tends to zero, so that the string coupling parameter $g_s = 1/\Phi$ diverges. This problem may probably be coped with due to the superstring dualities, mapping a strong coupling regime to a weak coupling regime. Such a difficulty is, however, absent in the above models interpolating between two type II asymptotics.

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